SERIES WITH POSITIVE MEMBERS (criteria)

Consider a number series $a_1 + a_2 + a_3 + ... + a_n + ... = \sum_{n=1}^{\infty} a_n$ with positive members.

The sum is $S_n = a_1+a_2+a_3+\ldots+a_n = \sum_{k=1}^{n} a_k$, and we call it **partial sum**.

We are looking for $\lim_{n\to\infty} S_n$.

If $\lim_{n\to\infty} S_n$ =S (number) then series **converges**, and if $\lim_{n\to\infty} S_n = \pm\infty$ or does not exist, then the series **diverges.**
This means that series converges when the sequence S_n of partial sum has a finite limit and if the limit of partial sum is infinitive or does not exist, the series diverges.

Cauchy criteria (test)

Necessary and sufficient condition for $\sum_{n=1}^{\infty} a_n$ to converges is that for arbitrary $\varepsilon > 0$, there is a natural number $N = N(\varepsilon)$ so that for $n > 0 \wedge p > 0$ is true : $\left|S_{n+p} - S_n\right| < \varepsilon$

Theorem: If the series $\sum_{n=1}^{\infty} a_n$ converges , then $\lim_{n\to\infty} a_n$ =0, if $\lim_{n\to\infty} a_n \neq 0$ then line certainly does not converge.

Comparable criteria:

Valid for two series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$

i) If $a_n < b_n$ then a) $\sum_{n=1}^{\infty} b_n$ convergent $\Rightarrow$ $\sum_{n=1}^{\infty} a_n$ convergent

b) $\sum_{n=1}^{\infty} a_n$ divergent $\Rightarrow$ $\sum_{n=1}^{\infty} b_n$ divergent

ii) If $\frac{a_{n+1}}{a_n} < \frac{b_{n+1}}{b_n}$ then a) $\sum_{n=1}^{\infty} b_n$ convergent $\Rightarrow$ $\sum_{n=1}^{\infty} a_n$ convergent

b) $\sum_{n=1}^{\infty} a_n$ divergent $\Rightarrow$ $\sum_{n=1}^{\infty} b_n$ divergent

iii) If $\lim_{n\to\infty} \frac{a_n}{b_n} = M$, ($M \neq 0$ and M is a finite number) series simultaneously are convergent or divergent

Most often series we used for comparison is $\sum_{n=1}^{\infty} \frac{1}{n^k}$; for k>1series is convergent , for $k \leq 1$ is divergent.

D`Alember criteria

If for series $\sum_{n=1}^{\infty} a_n$ there is $\overline{\lim_{n\to\infty}} \left|\frac{a_{n+1}}{a_n}\right| = r$ then :

- For r > 1 series is divergent
- For r = 1 undecidable
- For r < 1 series is convergent

Roots Cauchy criteria :

If for series $\sum_{n=1}^{\infty} a_n$ there is $\overline{\lim_{n\to\infty}} \sqrt[n]{|a_n|} = p$ then :

- For p > 1 series is divergent
- For p = 1 undecidable
- For p < 1 series is convergent

Raabe criteria :

If for series $\sum_{n=1}^{\infty} a_n$ there is $\lim_{n\to\infty} n(\frac{a_n}{a_{n+1}} - 1) = t$ then :

- For t > 1 series is convergent
- For t = 1 undecidable
- For t < 1 series is divergent

Gauss criteria

If for series $\sum_{n=1}^{\infty} a_n$ there is

$$\frac{a_n}{a_{n+1}} = \lambda + \frac{\mu}{n} + o(\frac{1}{n^{1+\varepsilon}}) \quad \text{for} \quad \forall \varepsilon > 0 \text{ then :}$$

i) If $\lambda > 1$ series is convergent
ii) If $\lambda < 1$ series is divergent
iii) If $\lambda = 1$ then $\begin{Bmatrix} \text{for } \mu > 1 \text{ is convergent} \\ \text{for } \mu < 1 \text{ is divergent} \end{Bmatrix}$

Cauchy integral criteria

If the function f (x) decreases, it is continuous and positive , then series $\sum_{n=1}^{\infty} f(n)$ convergent or divergent simultaneously with integral $\int_1^{\infty} f(x)dx$